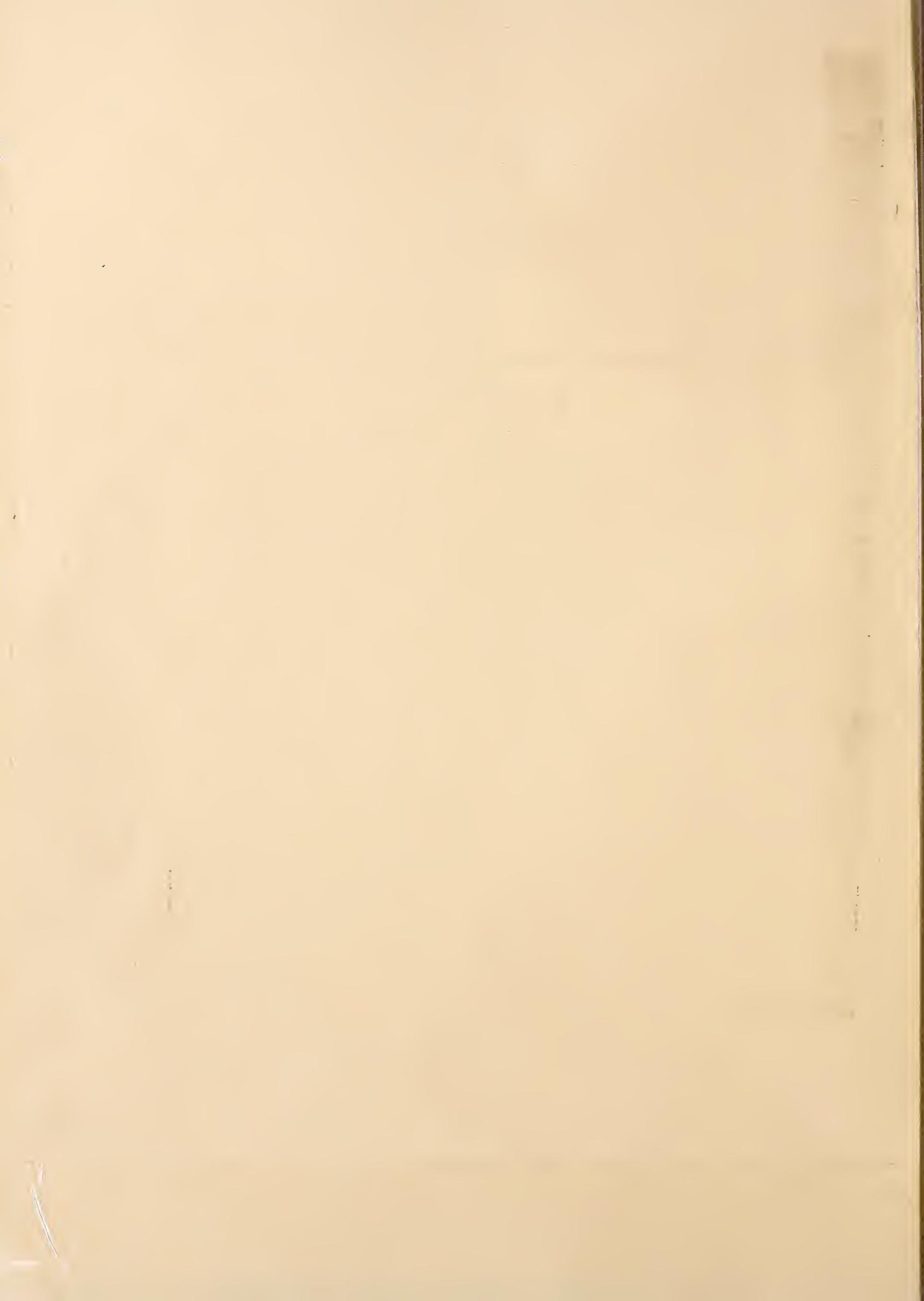


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PREDICTING PRODUCT RECOVERY FROM LOGS AND TREES

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MAY 21 1971

CURRENT SERIAL RECORDS

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INTRODUCTION

Value of logs or trees is judged by buyers and sellers on the basis of some estimate of the amount of lumber, veneer, pulp, or other product that can be made from them. This paper discusses estimating equations, based on size of logs or trees, that can be used for such predictions. Some of these have been used for many years; others are of recent origin. Formula log rules are familiar to foresters who studied mensuration in college; other predictors will be strangers to most.

This paper briefs some historical development of timber product recovery prediction equations and demonstrates relationships among various equation forms. It states statistical considerations important in the development of such predictions. It discusses primary units of tree measure—volume, surface, and length. It shows that use of these units leads to equations with certain limitations on their predictive ability. It points out that these independent variables may be used for estimating other quantities of interest related to log and tree size such as sawing time, hauling costs, or log weight. Finally, it presents estimating equations that recently have been developed to predict volume of Douglas-fir lumber and veneer and value of lumber. It should serve as both a refresher on the subject of formula-type board-foot log rules and an introduction to newer product recovery predictors. Some of this is so new that it is a progress report on unfinished research.

Although discussion is divided into five sections, there is a development of the interrelations among the sections. This development contains a great many equations because such estimates are based on equations. Most new developments are presented as one further step from a previous equation. Hopefully, such a step-by-step approach will clarify some of the many ramifications and implications of these estimators and their transformations.

Regular Formula Log Rules and Some Variations

Log rules are one of the oldest kinds of estimators of lumber recovery from logs. One of the oldest formula log rules still in use was published by Clark in 1906 (3). He pointed out that allowance must be made for shrinkage and waste in estimating the volume of square-edged boards that could be manufactured from logs. He also indicated that shrinkage and kerf allowances were in proportion to the cross sectional area of the log (proportional to the square of diameter or D^2) and that allowances for circular form of log, minor crook, and average sweep were in proportion to the bark surface area (or for a given length in proportion to diameter or D). His basic equation for board feet of lumber in a 4-foot section of log, assuming 1/8-inch kerf, is $B = 0.22D^2 - 0.71D$ (where B is board feet of lumber and D is scaling diameter in inches). In this, the coefficients (0.22 and 0.71) include the allowances he found suitable in studies of hardwoods and conifers in the Northeast. When this basic 4-foot equation is used for longer logs with an allowance of 1-inch-in-8-feet increase in diameter, the equation becomes $B = aD^2 + bD + c$ with the coefficients (a , b , and c) changing for each log length.

The Scribner log rule is not a formula log rule, but an equation was published in 1925 (1) that does a good job of duplicating Scribner estimates. This is based on the same equation form as one given in the last paragraph for the International rule. This equation also serves as the basis for several other log rules. Some coefficients for 16-foot logs are

<i>Log rules</i>	<i>a</i>	<i>b</i>	<i>c</i>
Scribner	+0.79	-1.98	-4.30
British Columbia	+ .76	-2.28	+1.71
International 1/4-inch	+ .80	-1.37	-1.23
Doyle	+1.00	-8.00	+16.00

To convert most of these to rules for logs of any length, it is necessary to divide the coefficients by 16 and put the unknown length (*L*) into the equation

$$B = a'D^2 L + b'DL + c'L.$$

In this form, the first term can be recognized as proportional to scaling cylinder volume, the second term to scaling cylinder lateral surface, and the third term to length.

Since the International rule allows for taper, the coefficients of its equation must be determined from the basic equations for 4-foot sections, first reducing the coefficients by dividing by four and then averaging the coefficients for the indicated number of 4-foot sections. A six-term equation written by Schumacher (10) does this for logs of any length. Some coefficients for logs of different lengths are

<i>Log rules</i>	<i>a'</i>	<i>b'</i>	<i>c'</i>
Scribner	+0.049	-0.124	-0.269
British Columbia	+ .048	- .143	+ .107
International 1/4-inch (16-foot logs)	+ .050	- .086	- .077
International 1/4-inch (32-foot logs)	+ .050	- .014	- .063

Log rules with such coefficients are useful for estimating board feet of potential lumber. If we want to compare volume of lumber with volume of log (the recovery ratio or percent recovery), we have to change these coefficients. The easiest change is to compare the volume of lumber with the volume of scaling cylinder (recognizing that this is an approximation since logs have slightly more volume than their scaling cylinders). The cubic-foot volume of the scaling cylinder (C_C) is

$$C_C = 0.00545 D^2 L.$$

Dividing this into the Scribner board-foot estimate, we get

$$\begin{aligned} B_S/C_C &= (0.049D^2 L - 0.124DL - 0.269L) / (0.00545D^2 L) \\ &= 9.06 - 22.7/D - 49.3/D^2. \end{aligned}$$

This estimates board feet of lumber per cubic foot of scaling cylinder. It can be changed to a rough estimate of recovery ratio by changing board feet of lumber to cubic feet of lumber. Dividing both sides of the equation by 12 does this:

$$B_S/12C_C = C_S/C_C = 0.75 - 1.89/D - 4.11/D^2.$$

This equation shows that Scribner formula rule estimates, converted to cubic feet of rough green lumber, are three-fourths or less of the total cubic volume of the scaling cylinder. Since the scaling diameter and its square appear in the denominators of terms with negative coefficients, the predicted recovery is lower for smaller logs. Predicted percentage converted to lumber of any log is less than this because the log is larger than the scaling cylinder.

If it is assumed that logs of a given length are frustums of either cones or paraboloids and an equation is written showing Scribner estimates of lumber recovery per cubic foot of the entire log (C_L), instead of just the scaling cylinder, the result will be

$$C_S/C_L = a + b/D + c/D^2 + d/D^3 + e/D^4 + f/D^5 + \dots$$

In this equation, the terms with the cube and higher powers of D enter because the log is tapered.¹ The effect of these terms is quite small.

A similar equation for International 1/4-inch estimates of lumber recovery for 16-foot logs, which assumes conical taper of 1 inch in 8 feet, divided by cubic volumes of the same shape logs is

$$C_I/C_L = 0.76 - 2.84/D + 4.53/D^2 - 4.49/D^3 + 3.02/D^4 - 4.03/D^5 + \dots$$

If the International equation for logs of all lengths (10) is divided by the cubic volume of conical "logs" of the same lengths, the first two terms stay the same but succeeding terms have powers of L in their numerators.

These equations, expressing lumber recovery in cubic feet as a fraction of log cubic volume, can be changed back to estimates of recovery in board feet of lumber per cubic foot of log by multiplying all coefficients by 12.²

There are two things I want to emphasize about these formula log rules: (1) Board-foot log rules are estimates of lumber recovery. The numbers on scale sticks are units of estimate, not units of measure. (2) When we express lumber recovery as cubic feet of rough green lumber per cubic foot of log, we are well on our way to accounting for all the fiber in the log. When we also compute or measure the cubic feet of sawdust per cubic foot of log, we can estimate the cubic feet of chippable residue.

¹The equation for the volume of the frustum of a cone, where $(D_L - D_S)/L = a$ (i.e., a is increase in diameter per foot of length when D_S is small end diameter and D_L is large end diameter) is

$$V = \pi L (D_S^2 + aD_S L + a^2 L^2/3)/4.$$

For logs of a given length, this reduces to

$$V = bD_S^2 + cD_S + d$$

which gives the series indicated when divided into the Scribner equation. Similarly, the equation for a frustum of a paraboloid of a given length reduces to

$$V = eD_S^2 + f$$

which gives a similar series.

²This assumes the dimensions of rough green lumber are the nominal dimensions. Other assumptions about dimensions require conversion factors ranging from about 11 to 13.

Empirical Log Rules and Batch Studies

In 1940, Schumacher and Jones reported a study of empirical log rules (10). This report includes three important principles.

The first principle is that a rational and useful algebraic form for estimates of lumber recovery (B) in mill studies is

$$B = aD^2 L + bDL + cL.$$

This three-term equation is the same as the formula log rule. This means that such log rules are no more empirical than the log rules just discussed since they use the equation based on Clark's theoretical work. What distinguishes them is that they rely on least squares fitting of the equation to observed data.

The second principle is that to get least squares estimates, unaffected by unequal variances, this equation must be weighted. Schumacher stated "... As volume is a function of the product of linear dimensions, it is to be expected that the standard deviation of volume of single logs ... is proportional to $D^2 L$." He demonstrates this by showing that a sloping straight line fits the standard deviations of volume of 820 logs divided into 30 size-classes and plotted over size-class. Statistically, this requires the sum of squared residuals to be weighted by the reciprocals of variances. This is accomplished by dividing both sides of the equation by $D^2 L$. When the dependent variable is divided by $D^2 L$, we get for a new dependent variable a value proportional to board feet of lumber divided by volume of scaling cylinder. We could further scale the variables by dividing them all by 0.005454 in which event our "transformed" dependent variable would be ratio of board feet of lumber to cubic feet of scaling cylinder:

$$B/0.005454D^2 L = B/C_C.$$

Since dividing all observed values by a constant has no effect on the coefficients derived by least squares, this step is unnecessary when processing data. It is only useful if we want to look at observed values of board feet of lumber per cubic foot of scaling cylinder.

Dividing by $D^2 L$ also gives us dimensionless ratios for dependent variables that do not have built-in high correlations with the independent variables. Such high correlations sometimes lead to acceptance of functions that do not estimate as well as those derived from analysis of dimensionless ratios. There appears to be a tendency to accept oversimplified functions that have high correlations.

The third principle that Schumacher discussed is the suitability of the basic algebraic relation for analysis of batch data. This idea was suggested by Sir Ronald Fisher in 1936 at a seminar in Asheville, N.C. The problem was allocation of transportation costs to logs of different sizes when there was a fixed rate (Y) for each full carload. The equation for each carload he suggested was

$$Y = a\Sigma D^2 + b\Sigma D + cN$$

where Σ is sum for each carload and N is number of logs for each carload. This was to be fitted for all observed carloads. This batch equation has been used by Day and by Hasel in logging cost problems (5, 8), by Chisman and Schumacher and by Lexen in stand density studies allocating areas to trees of different sizes based on plot summaries (2, 9), and by Schumacher and Jones in the study of empirical log rules I have already mentioned (10).

Other uses that are appropriate are studies of log weights based on truck weighing and studies of the effect of log size on sawing time based on shift totals.

The principle involved in such studies is that where only the total Y is observed it represents the sum of individual y 's for each member of the batch, group, or plot. Where each individual y is highly correlated with the size of the same individual, and the relation can be expressed in an equation with linear coefficients, analysis of sets of data using only totals will yield equations that predict individual y 's.

It should be apparent that Fisher's equation

$$Y = a\sum D^2 + b\sum D + cN$$

would be rewritten if there were much variation in log length as

$$Y = a'\sum D^2 L + b'\sum DL + c'\sum L.$$

This is the unweighted form of the equation that Schumacher and Jones fit to data on logs used and lumber produced on each of 18 days at a small sawmill. They compared the standard error of their batch equation with a standard deviation of volume from the 820 individual observations and estimated that 23-day totals would have been required for the same amount of information on single saw logs that was contained in direct measurement of 820 individual saw logs.

Cubic Volume, Surface Area, and Length Are Primary Units of Tree Measure

In 1954, Grosenbaugh showed that log total cubic volume, surface area, and length could be substituted for corresponding measures of the scaling cylinder in log rules (6). Several years later, he described the use of these primary units in product recovery studies and for other purposes (4).

The benefits of this change in variables are apparent in tree estimates. Each tree has but a single total merchantable cubic volume, surface area, and length. Repeated careful measurements should be extremely close to these totals. Each tree can have many different $\sum D^2 L$ and $\sum DL$ since there are many different ways to buck the tree and each $D^2 L$ and DL is based on a particular scaling cylinder. Further, the total volume of the scaling cylinders is less than the merchantable volume of the tree. When a tree is a candidate for more than a single use, it is convenient not to have to generate separate sets of scaling cylinder dimensions for saw logs and for veneer blocks.

In some trees, all logs are in a single stratum of quality and defect.³ In other trees several strata of quality and defect can be recognized. Fisher's batch equation appears suitable for use on trees or on strata within trees. Because log dimensions within each tree are progressively smaller the coefficients of estimating equations differ somewhat from those based on random batches of logs.

Grosenbaugh also has showed how weight can be substituted for cubic volume as a primary unit of log measure (7). Truck scales make this one of the quickest ways to measure logs. Total length of logs on the load should also be recorded so that these two

³ It is assumed that external indicators exist that suggest changes in value of product (quality) and changes in volume of product (defect).

measures can be substituted in product estimating equations for volume and length. Generally the weight/volume ratio for a given class of logs does not vary much, so estimators based on weight will be as accurate as those based on volume. Obviously the weight/volume ratio varies with species, is different for small cold decked logs than for those measured soon after cutting, and may also vary with log diameter. But these sources of variation also require different product estimating coefficients. If it is feasible to aggregate the square root of the product of weight and length, this aggregate figure can be substituted for surface area in estimating equations.

The idea that surface area is a unit of log or tree measure seems hard for some people to grasp. Most are quite ready to accept length and cubic volume as units of measure—length is a basic unit and volume is a derived unit, the product of three lineal dimensions. When a surface is flat, it is customary to measure it in acres, square feet, or square inches—the product of two lineal dimensions. The log surface we are interested in is the inside bark surface which excludes the end surfaces. It can be visualized as a flat sheet wrapped around a solid whose shape is almost cylindrical.

For cylinders measured in feet of length and inches of diameter, we have

$$\begin{aligned}\text{Cubic volume: } C(\text{cubic feet}) &= D^2 L / 183.3 = 0.005454 D^2 L. \\ \text{Surface area: } S(\text{square feet}) &= DL / 3.819 = 0.2618 DL. \\ \text{Length: } L(\text{feet}) &= L.\end{aligned}$$

The equations for frustums of cones and paraboloids, which usually approximate log shapes more closely than cylinders, are only a little more complicated. They may be based on length and diameters of both ends, and each can be related to diameters of the same length cylinder having equal volume or surface. Diameters of these cylinders are about midway between the two end diameters of the frustums and can be called equivalent cylinder diameters (D_E).⁴ My reason for mentioning this comes from consideration of the equation expressing cubic feet of lumber (Y)

$$Y = aC + bS + cL.$$

When this is weighted by dividing by C (the equivalent of $D^2 L$ in previous equations), it becomes

$$Y/C = a + bS/C + cL/C.$$

This can be rewritten in terms of equivalent cylinder diameters:

$$Y/C = a + 0.26 bD_E L / 0.0054 D_E^2 L + cL / 0.0054 D_E^2 L$$

which can be simplified to

$$Y/C + a + b'/D''_E + c'/D'_E^2$$

where D'_E and D''_E are slightly changed values of D_E .

The next step is to extend this concept of equivalent diameters from logs, where they approximate mid-diameters, to trees. In trees the diameters (and squared diameters) corresponding to $(\Sigma C / \Sigma S)$ and $(\Sigma C / \Sigma L)$ are weighted averages, not simply half the basal

⁴The equivalent cylinder diameter based on S is not identical to that based on C .

diameters or their squares. The ratios of these sums include the information on average log size that is needed to make good estimates of product recovery for trees.

Another way of viewing this is that when we use C , S , and L in a weighted analysis we are really fitting a function that includes reciprocals of a diameter and a squared diameter. In most of the rest of this discussion, I shall use only this last form of the equation or a closely related one based on cubic volume and scaling diameter.

When the weighted product recovery equation

$$Y/C = a + bS/C + cL/C$$

is fit to data from trees with two strata of quality or defect, it may be written in the form of a batch equation

$$Y_T/C_T = aC_1/C_T + bS_1/C_T + cL_1/C_T + dC_2/C_T + eS_2/C_T + fL_2/C_T$$

where the subscripts

- T is individual tree total,
- 1 is stratum 1 total in each tree, and
- 2 is stratum 2 total in each tree.

With this form of the equation, the individual sections that produce each board or each sheet of veneer are not identified and are not necessarily the same as the sections recognized in the tree. A tree that has more of its volume in high quality sections than another tree will produce a greater proportion of high quality products. Where Y_T is total product value, it will tend to be higher even when the high quality tree section is less than a full log in length. The efficiency of this form of the equation is being tested.

The coefficients of this equation depend not only on how the tree is bucked but also on the efficiency and product objectives of the mill. It should be apparent that each set of coefficients will serve only for the mill at which it was produced. Area averages will be based on several studies which include different bucking rules and different mill objectives and efficiencies.

Characteristics of Equation $Y = a + b/D + c/D^2$

Since both the theoretical log rule equation and the tree product equation based on volume, surface, and length are transformed by weighting to the same form, it seems appropriate to examine some of the characteristics of this equation. This will give insight into why the equation works and also into some of its limitations.

Figures 1 and 2 show different families of curves that are generated by holding two of the coefficients constant and requiring the third coefficient to be either positive or negative. Figures 1A and 1B have the coefficient of one of the reciprocal terms set at zero and the other negative. These families show lower Y values as D gets smaller. The rate at which Y decreases depends on whether the coefficient that varies belongs to the D or D^2 term. Figure 1C, with both coefficients of the reciprocal terms negative, is intermediate in shape between figures 1A and 1B. It is easy to visualize the effects of positive coefficients. The families of curves would be symmetrical around the line $Y = 0.7$ to those plotted. However, all curves as plotted show the decline in recovery ratio that is typical of small diameter logs in sawmills or veneer mills. Figure 1D is included to show

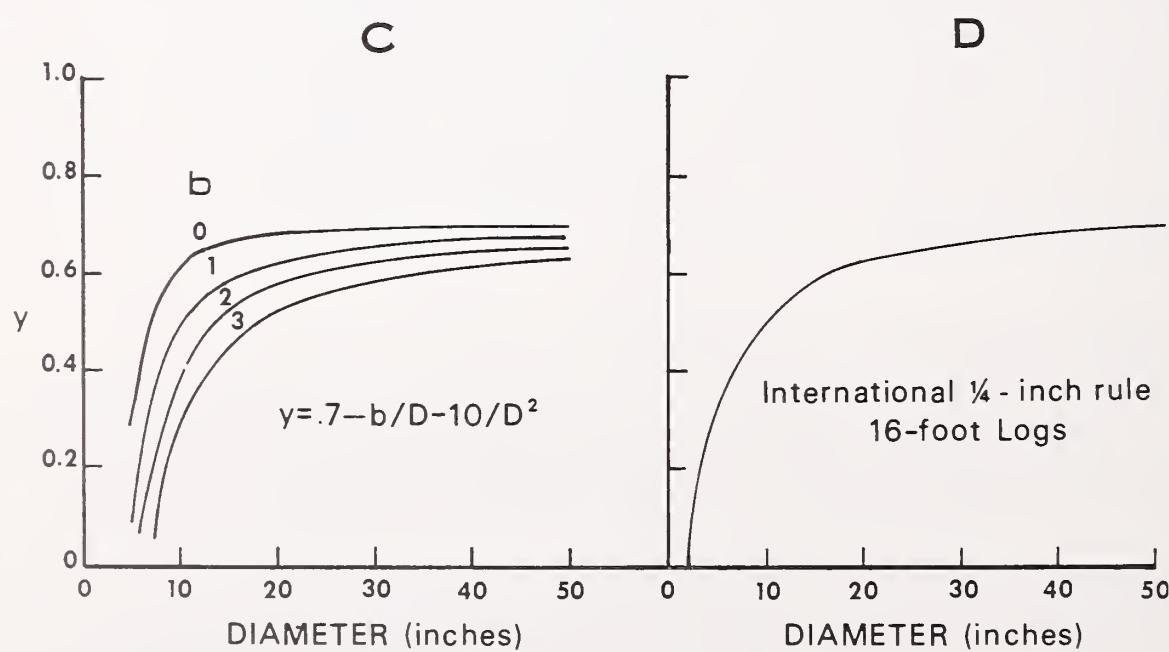
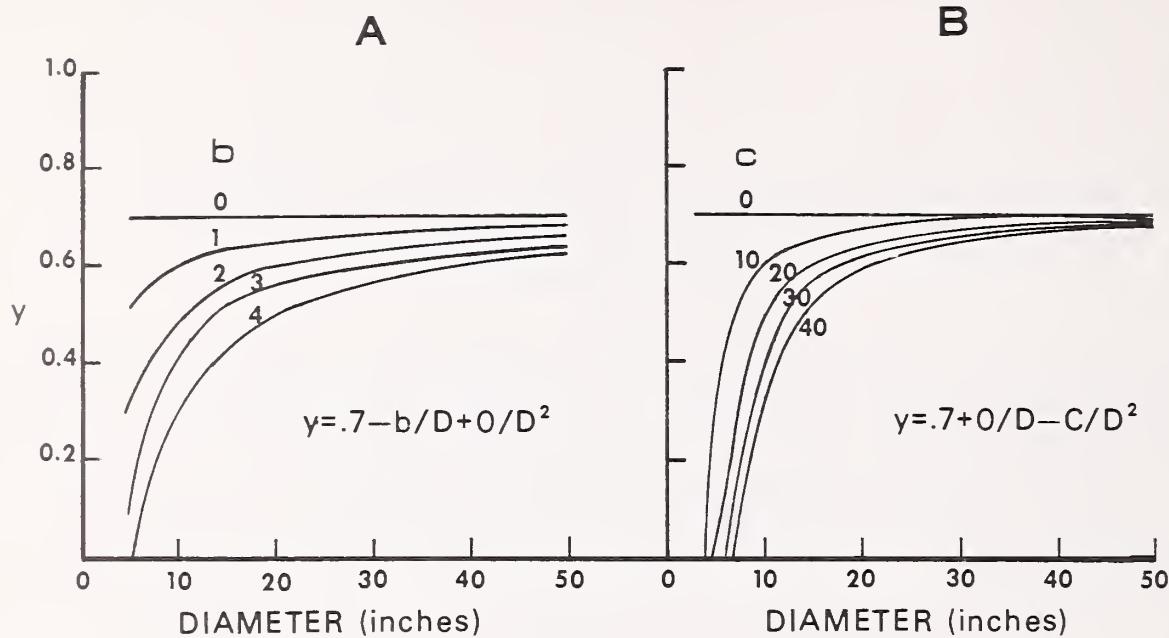


Figure 1.—Families of curves: $Y = a + b/D + c/D^2$.

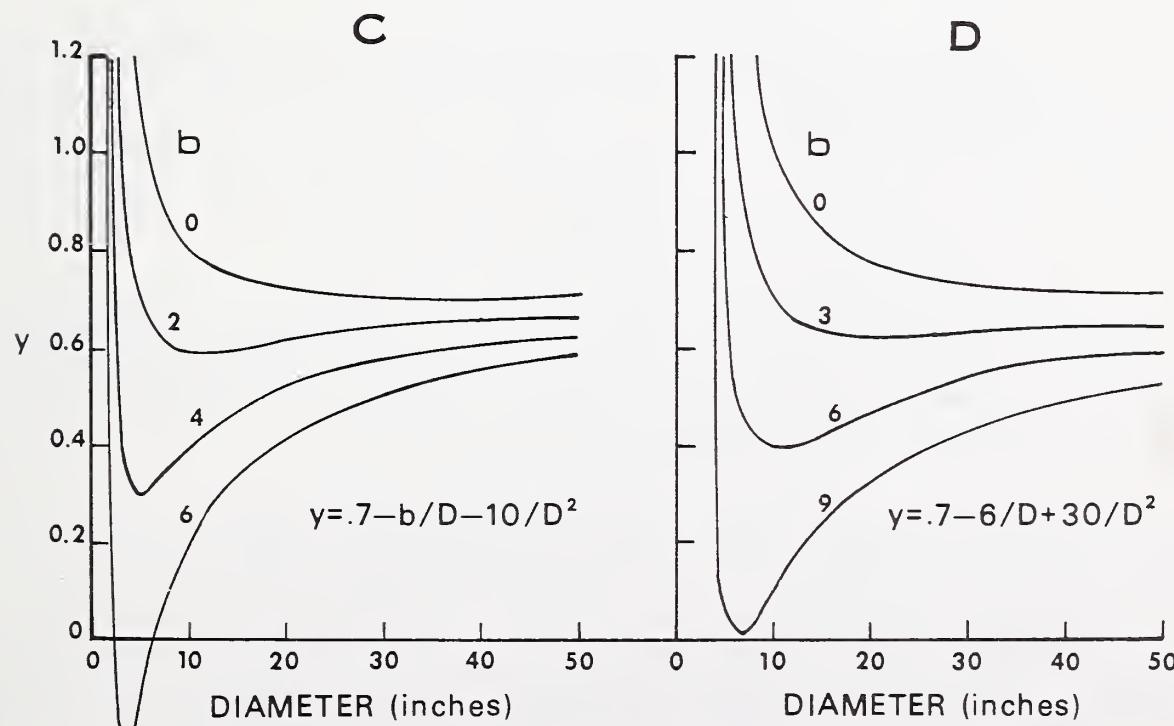
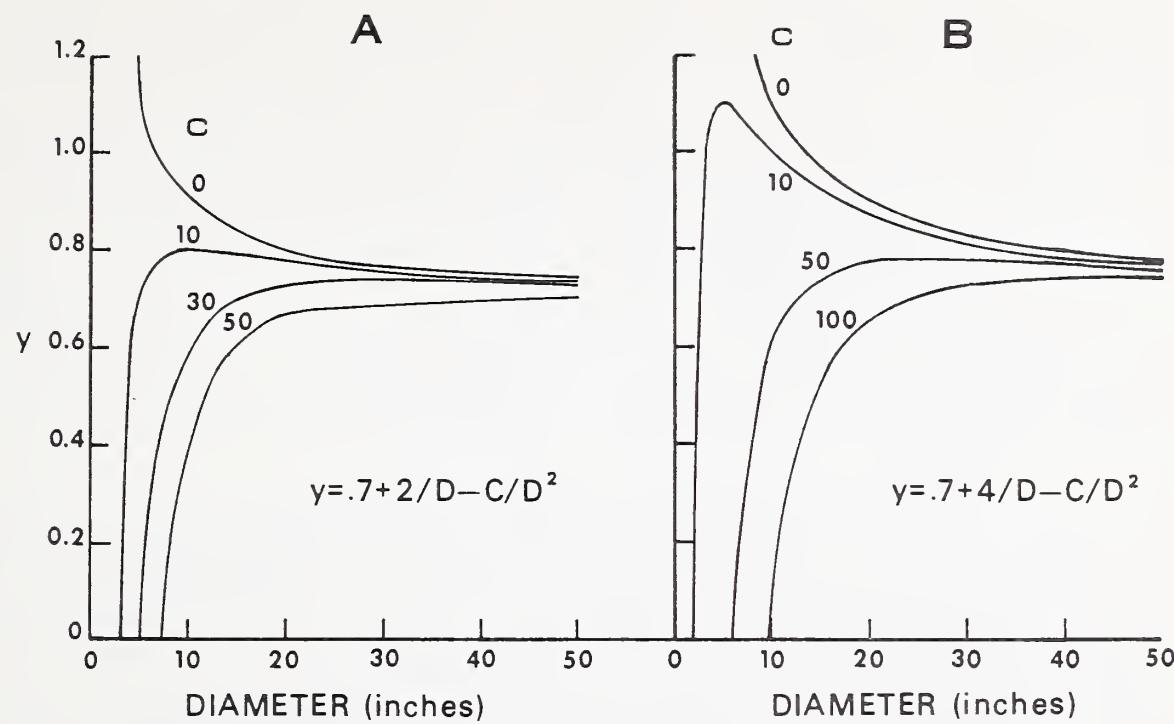


Figure 2.—Families of curves: $Y = a + b/D + c/D^2$.

that the family of curves in figure 1C describes the lumber recovery ratio estimated by the International rule.

Having looked at some well behaved curves belonging to the family, the reader should consider some that appear erratic. Figure 2 illustrates some of the families generated when the coefficient of one of the reciprocal terms is positive and the other negative. Mirror images of these (rotated around $Y = 0.7$) can be produced by multiplying the coefficients of the second and third terms by -1. When such equations fit product recovery data, it usually is essential that the equations not be used for small logs and may be necessary to restrict any use of the equation to all the logs or trees from which they were derived. These restrictions seem much less important for the families of curves plotted in figure 1.

Some of these curves, such as the upper two in figure 2B, may fit data from a batch of large logs (none under 18 inches) with defect increasing as diameter increases. Others, such as the third and fourth curves in figure 2D, may fit data where the recovery ratio or other dependent variable drops markedly between 40 and 20 inches. The shape of this last curve suggests that other functions might better fit the data.

If in fitting these weighted equations either of the coefficients of the last two terms is not negative or close to zero, the estimates should be plotted over diameter to see whether estimates for small diameters are reasonable. With negative coefficients, unreasonable estimates are less likely, but it is still possible to get estimates of zero recovery for 7- or 8-inch logs or positive recovery for 3- or 4-inch logs. This happens because mill recovery studies seldom include logs whose diameter reduces recovery to less than about 35 percent.

When solving equations in volume, surface, and length, it is convenient for plotting purposes to assume 1 inch of taper in 8 feet and reduce equivalent diameters (middle diameters) to small end diameters.

Product Recovery Equations for Lumber and Veneer

These equations have been fit to data gathered by this Station's Grade and Quality of Western Timber Project at three mills. The coefficients presented are for illustrative purposes only, and their use should be limited to this purpose or to the mill and assumptions for which they were derived.

Figure 3A, showing cubic volume of lumber recovery as a ratio to volume of log, demonstrates similarities among three mills. The equations on which these curves are based are

1. $Y/C = 0.660 - 1.31/D - 0.182Z$ (solid line) $(N = 445 \text{ logs})$.
2. $Y/C = 0.627 - 0.95/D - 0.270Z$ (dashed line) $(N = 360 \text{ logs})$.
3. $Y/C = 0.649 - 0.01/D - 10.7/D^2 - 0.077Z$ (dotted line) $(N = 561 \text{ logs})$.

Similar equations that give nearly identical estimates based on log volume, surface, and length, instead of log volume and scaling diameter, are

1. $Y/C = 0.661 - 0.031S/C - 0.187Z$.
2. $Y/C = 0.623 - 0.020S/C - 0.205Z$.
3. $Y/C = 0.626 - 0.018S/C - 0.109L/C - 0.067Z$.

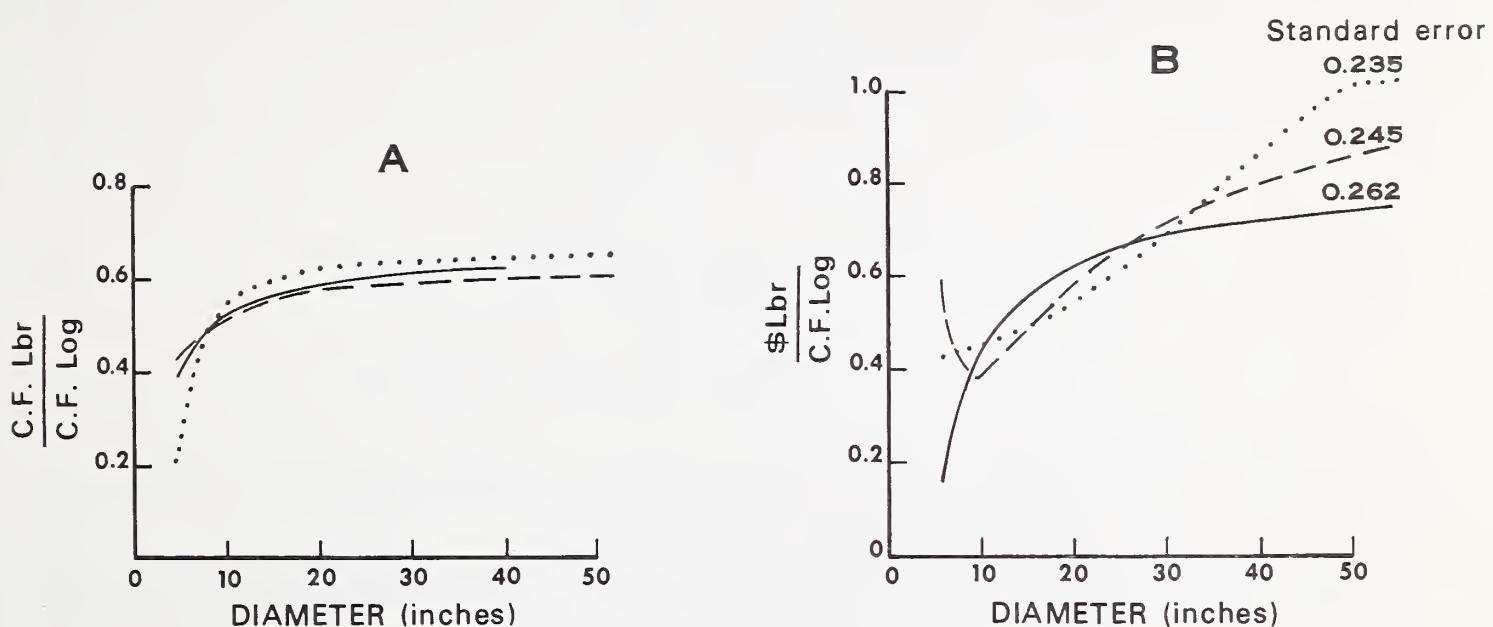


Figure 3.—Cubic feet of Douglas-fir lumber per cubic foot of log
for three sawmills in Oregon and Washington.

To convert these equations to direct estimates of product volume, both sides should be multiplied by C . The last equation becomes

$$Y = 0.626C - 0.018S - 0.109L - 0.067ZC.$$

The Z in these equations is the defect ratio for those defects visible in the standing tree, i.e., the scaled defect deduction divided by the gross scale. The graphs show recovery for logs with average Z . Sound logs had only 0.01 greater recovery. Had all defects, including those visible on ends of logs, been used, this Z term would have had a larger effect.

The most effective weighting factor to use in these analyses appears to be cubic feet of log, not cubic feet of scaling cylinder or $D^2 L$. In fitting of such weighted equations, the independent variables that usually have the smaller mean squared residuals are reciprocals of scaling diameters or their squares rather than the ratios L/C and S/C . These equations may provide the basis for judging the efficiency of other analyses of the same data either as individual logs or as entire trees.

When product value is substituted for product volume as dependent variable, the irrational curves previously mentioned sometimes result (fig. 3B). The dashed curve (based on $1/D$ and $1/D^2$) fits the data better than the solid curve (based on $1/D$). However, the lower end of this dashed curve is not acceptable as an estimate of value of small logs.

This strange curve can be explained. Dollar value of lumber per cubic foot of log is the product of lumber value for logs of different sizes and the ratio of lumber volume to log volume illustrated in figure 3A. If the range of lumber values from Select to Economy is 4 to 1, we can picture an average value per board foot falling steeply with log diameter from large clear peeler to limby upper logs.

The family of curves

$$Y = a + b/D + c/D^2$$

includes such rapid declines from 40 or 50 inches to 20 inches only in curves illustrated in figures 2C and 2D. These curves do not describe a steeply falling value in the middle and values that change gradually at the ends of the data.

The standard errors shown in figure 3B show the improvement in estimates from another function (dotted line). This is

$$Y = 0.383 + 2.72 \times 10^{-4} D^2 - 1.35 \times 10^{-9} D^5 - 0.490Z.$$

To estimate veneer recovery, I diagrammed a log to show how it might be converted to veneer (fig. 4).⁵ This required several assumptions. The most important of these was

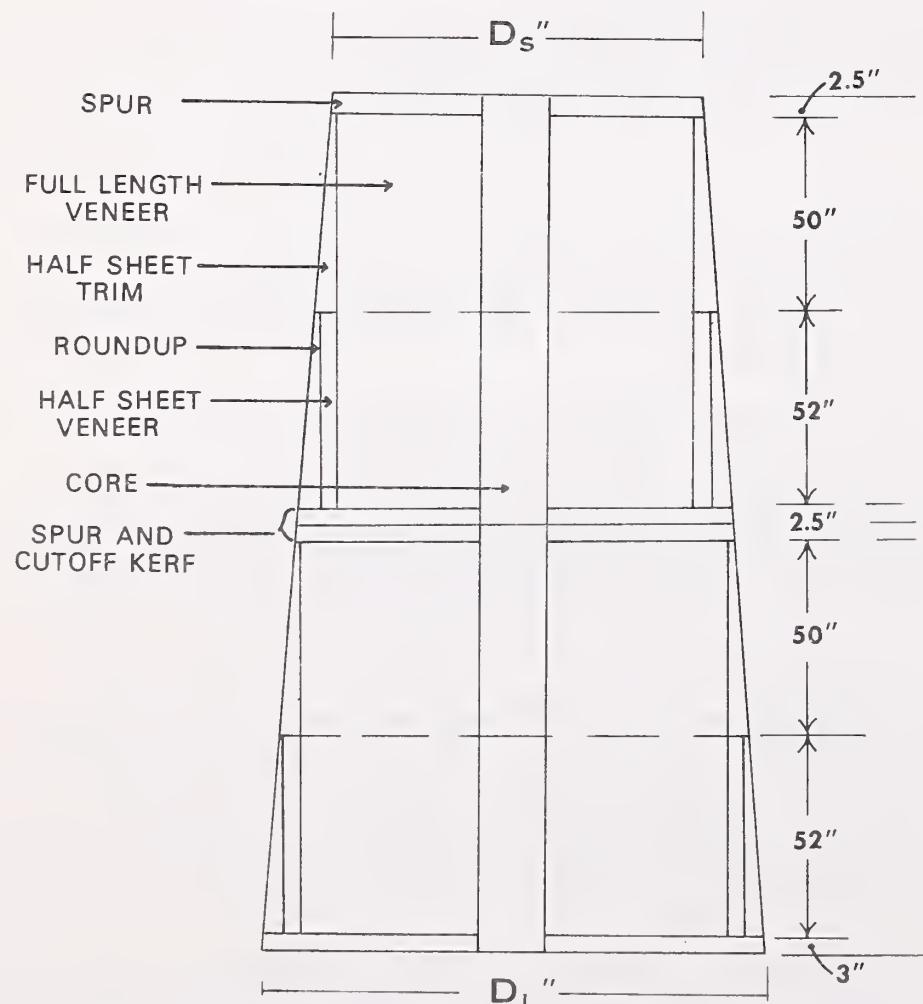


Figure 4.—Calculation of volumes in veneer logs (17 feet, 8 inches).

⁵ Richard O. Woodfin, Jr., in the Station's Grade and Quality of Western Timber Project, provided information about veneer production and the approximate dimensions involved.

that all logs get a small diameter deduction for surface roughness and that out-of-round logs get a deduction proportional to their diameter. These two deductions are for the material wasted before a full ribbon of veneer falls on the table. This diagram (fig. 4) was used to estimate green volume allocated to the veneer and several byproducts (assuming conical taper). These estimates are the basis for figure 5 which shows theoretical veneer recovery. Figure 5 shows that the diameter related losses due to taper and out-of-round may be nearly balanced by losses due to spur, kerf, and veneer shrinkage. Figure 5 can be compared with figure 6 which illustrates the recovery observed at one mill. The estimating equations for figure 6 are

Dry veneer	$y_1/C = 0.692 - 76.4/D^2 - 0.727 Z.$
Core	$y_2/C = 0.031 + 72.4/D^2 + 0.022 Z.$
Chippable waste and shrinkage	$y_3/C = 0.277 + 4.0/D^2 + 0.706 Z.$

These equations are based on 193 8-foot blocks with an average recovery ratio of 48 percent and average scaled defect of 1.0 percent. This defect deduction is based on log scaling. Note the closeness of the dashed line on figure 5 to the estimates for sound logs on figure 6. Also note that the sum of the y intercepts is 1.000 and the sums of the coefficients are close to 0. This shows that all the volume of the log is accounted for by the three estimates.

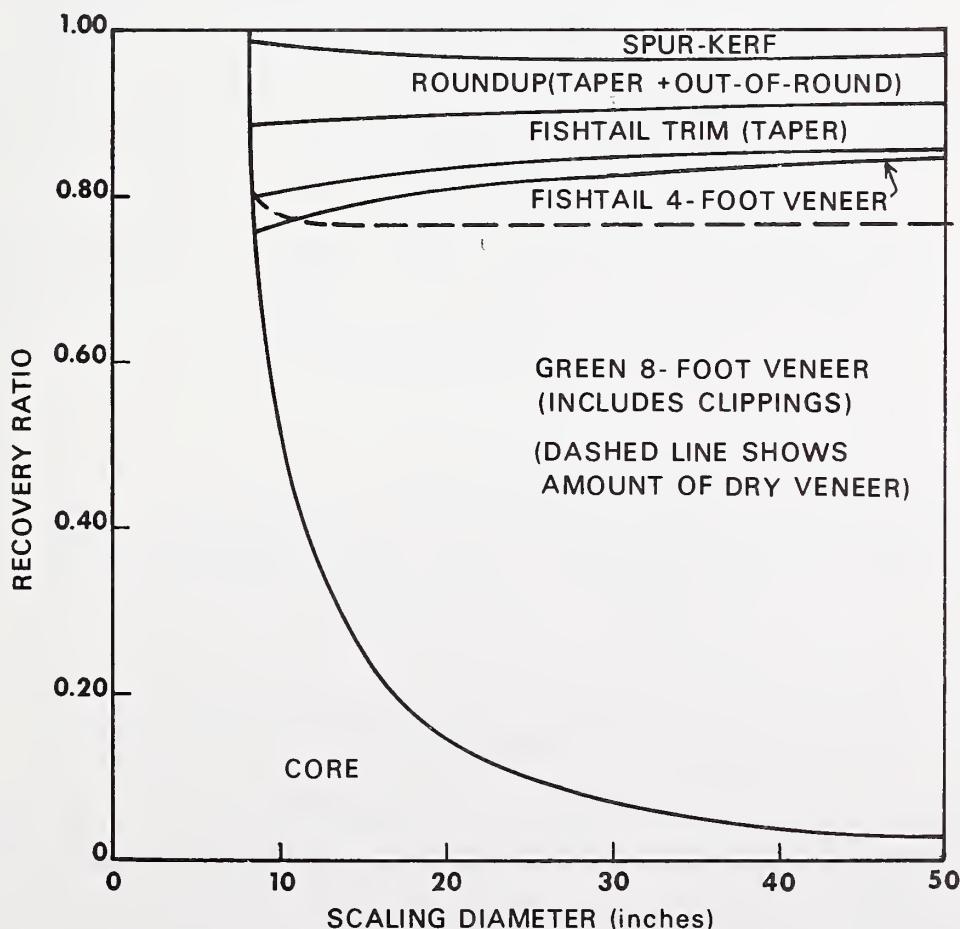


Figure 5.—Theoretical veneer recovery.

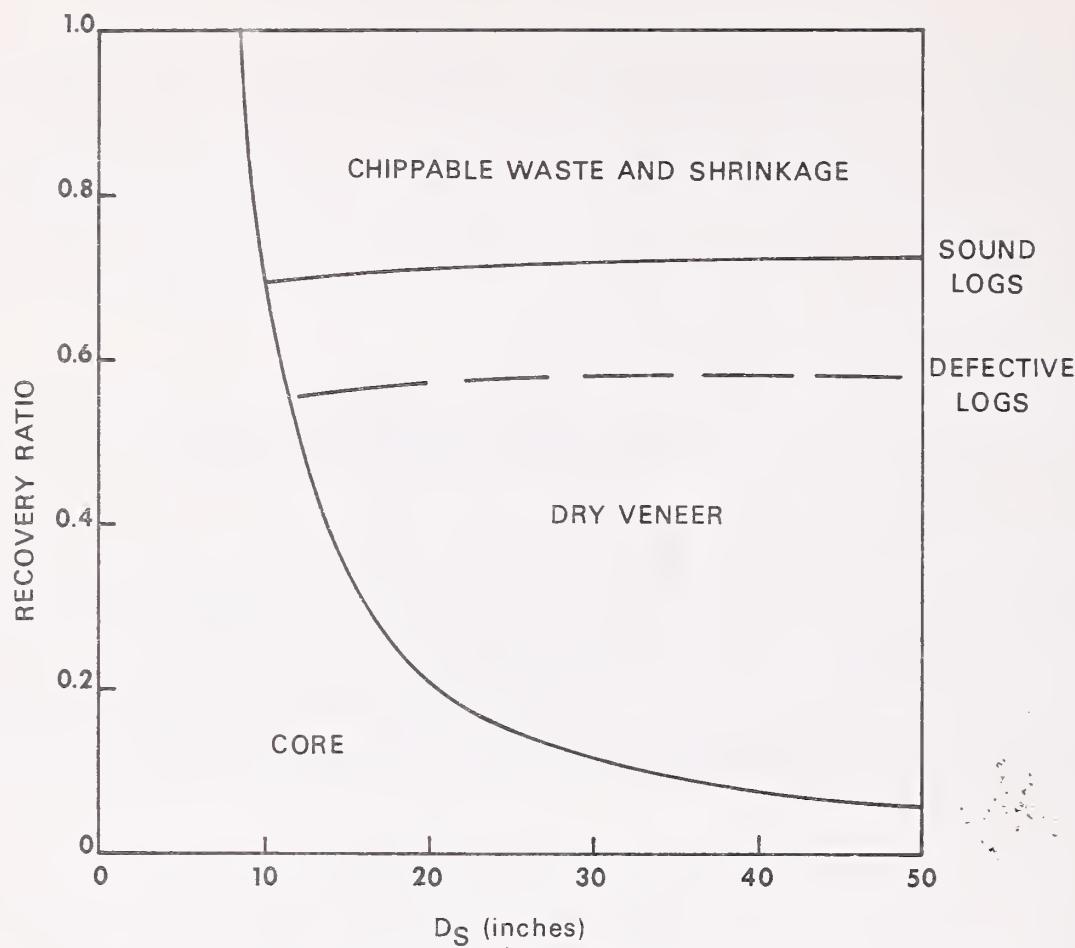


Figure 6.—Estimated veneer recovery based on observations at one mill.

SUMMARY

A theoretically based equation that has been used for formula log rules appears suitable for product recovery estimates

$$Y = aD^2L + bDL + cL$$

or, in another form,

$$Y = aC + bS + cL.$$

It has been shown that it usually should be fit by least squares as a weighted relation

$$Y/D^2L = a + b/D + c/D^2$$

or

$$Y/C = a + bS/C + cL/C.$$

Occasionally, these equations give wild estimates for small diameters, but such equations can be avoided.

There are benefits from using three units of tree measure (volume, surface, and length) rather than summations of measurements of scaling cylinders. A major one is the use of a single set of tree measurements to estimate different potential products. This same set of measurements can be obtained repeatedly by different observers.

There is no requirement that the quality or defect sections measured on the tree be identical with those into which the tree is bucked. The equation for two quality-defect strata within trees is

$$Y_T/C_T = aC_1/C_T + bS_1/C_T + cL_1/C_T + dC_2/C_T + eS_2/C_T + fL_2/C_T.$$

These equations have been tried for lumber and veneer recovery and work well for both.

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The mission of the PACIFIC NORTHWEST FOREST AND RANGE EXPERIMENT STATION is to provide the knowledge, technology, and alternatives for present and future protection, management, and use of forest, range, and related environments.

Within this overall mission, the Station conducts and stimulates research to facilitate and to accelerate progress toward the following goals:

1. Providing safe and efficient technology for inventory, protection, and use of resources.
2. Development and evaluation of alternative methods and levels of resource management.
3. Achievement of optimum sustained resource productivity consistent with maintaining a high quality forest environment.

The area of research encompasses Oregon, Washington, Alaska, and, in some cases, California, Hawaii, the Western States, and the Nation. Results of the research will be made available promptly. Project headquarters are at:

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Bend, Oregon	Olympia, Washington
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